C 207; Project Set #1

due thursday, 2/07/2013

Reflections on an Ancient Supernova

In 1572, Tycho Brahe noted the appearance of a new star ("nova stella"). Although he probably wasn't the first to see it, Tycho gets credit for the discovery, which he wrote about it in a book with the catchy title *"De nova et nullius aevi memoria prius visa stella"* ("Concerning the New Star, never before seen in the life or memory of anyone"). Actually it was a supernova that went off in our own Galaxy, only $D \approx 3$ kpc away. We can still see X-ray emission from Tycho's supernova remnant as it rams into the interstellar medium. The interaction emission, however, is complex and difficult to interpret, and people have debated exactly what kind of supernova Tycho had seen (unfortunately, he didn't take a spectrum of it, since spectroscopy wasn't invented yet).

Remarkably, it turns out that we can glimpse the same optical light that Tycho once saw. Some of the transverse radiation from the supernova scattered off of dusty clouds in the vicinity, and into our line of sight. Given the light travel time delay, this scattered light is just reaching earth now. Here we'll try to estimate the brightness of such a "light echo", mainly as an excuse to practice some basic radiative transfer. As a very simple model, we'll assume that both the supernova and the dust cloud are spheres of radii $R_{\rm sn}$ and R_c, respectively. The geometry is shown is Figure 1.

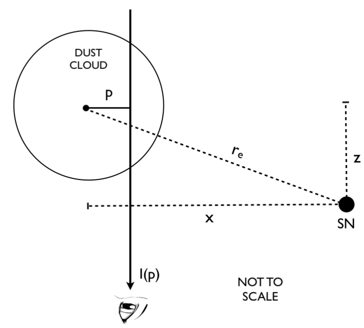

Figure 1: Diagram of the supernova light echo geometry.

a) If Tycho's supernova was a normal Type Ia supernova, it would have reached a luminosity of $L_{\rm sn} \approx 10^{43}$ ergs s^{-1}. Assuming that it emits like a blackbody at optical wavelengths ($T_{\rm sn} \approx 10000$ K), estimate the supernova radius around its peak brightness. How does $R_{\rm sn}$ compare to typical stellar sizes?[1]

[1] The natural explanation for the large size of the "new star" is that the stellar debris is expanding. Given the supernova rises to a peak in 20 days, you can easily make a good estimate of the expansion speed.

b) Assume that the supernova emits as a uniformly bright "Lambert sphere" – i.e., every point on the surface emits the same intensity, $I_0 = B_\nu(T_{\rm sn})$, in all outward directions. Show that mean intensity at a distance r from the center of the supernova can be written $J_\nu(r) = W(r)I_0$, and determine $W(r)$, known as the *dilution factor*.[2] This is a pretty fundamental and very useful result. Check the expression for $J_\nu(r)$ in the two limits $r \gg R_{\rm sn}$ and $r = R_{\rm sn}$ and explain why they make sense.

[2] For this part, it may be useful to adopt a geometry where r is along the z-axis.

c) Integrate J_ν over all frequencies and write the result, J, in terms of the supernova luminosity, $L_{\rm sn}$. Consider the limit $r \gg R_{\rm sn}$.

d) [Rest et. al 2008](#) imaged the surroundings of Tycho's supernova remnant and reported the discovery of several echoes. Consider the echo they found in field 4523, measured in the image to be 4.6 degrees away from the remnant center (see their table 1). Determine the distance, r_e, between the center of the dust cloud and the supernova. While you're at it, show that at a specific time all echoes lie on a parabolic surface ($z = ax^2 + b$), as illustrated in Figure 2. It is pretty cool that we can determine the full 3D position of an echo, just by knowing the time delay and the projected distance.

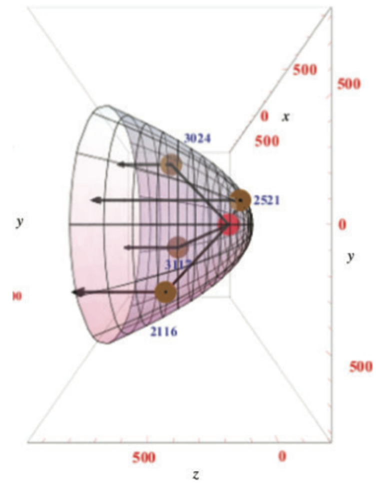

Figure 2: Visualization of some of the light echoes that have been found around a historical supernova (in this case, Cas A, not Tycho). You can tell something is an echo of the supernova because the emission moves overs time, as the paraboloid grows. In reality, the observed echoes usually look like arcs along the parabola, not spheres, but we will stick with a spherical approximation.

e) Let's assume that the dust cloud is relatively small ($R_c = 1$ light day) and optically thin enough that we can make the single scattering approximation (i.e., light scatters at most once in the cloud, and most of the light does not scatter at all). Then the mean intensity J_ν, due to the impinging radiation, is roughly constant throughout the volume. Assuming the scattering is isotropic, integrate the radiative transfer equation to determine the brightness profile of the echo – i.e., the specific intensity $I_\nu(p)$ as a function of the impact parameter p as viewed at infinity.

f) The observed light echoes can often be resolved, in which case we can measure the brightness profile. However, if we assume our cloud is small and unresolved, we would just measure the flux from a point source. Integrate the specific intensity $I_\nu(p)$ over the entire projected area of the cloud to determine the monochromatic flux F_ν of the light echo as detected on earth ($D \gg R_c$).

g) Since the cloud is optically thin, its monochromatic luminosity, L_ν can also easily be determined by integrating the emission coefficient over the entire volume. Show that calculating the flux as $F_\nu = L_\nu / 4\pi D^2$ (as you learn in Astro 10) gives the same result as in the previous problem.

Comment: While the Astro 10 way of calculating the observed flux seems a lot simpler, you should appreciate that it is only correct in the special case of an *isotropic* emitter. Often a source is not isotropic (imagine if the cloud was optically thick, and perhaps non-spherical). The general way to calculate F_ν is by integrating the specific intensity.

h) Write down the ratio of the flux of the light echo to the flux that Tycho would have observed from the supernova itself. If the optical depth of the cloud is $\tau \approx 0.1$, about how much dimmer is the echo than the supernova? You can consider $r_e \gg R_{\rm sn}$.

Comment: A proper analysis of light echoes is more involved, with a more complex geometry. The dust is actually distributed in extended filaments and/or sheets (not small spheres) and one needs to determine the intersection of these sheets with the paraboloid. You also have to fold in issues of seeing and the time evolution of the supernova light, see Rest et. al. (2011) .

A Problem of Great Magnitudes

In the old days, astronomers like Tycho classified the brightness of celestial objects using the magnitude system, which roughly corresponded to the sensitivity of the human eye. Today we continue to use this ancient system because... wait, why? In any case we have to get used to it. The magnitude of an object in a given filter i is

$$m_i = -2.5 \log_{10}[F_i/F_{0,i}] + m_{0,i} \tag{1}$$

where F_i is the observed flux integrated over a given filter

$$F_i = \int_0^\infty F_\nu(\nu)\phi(\nu)\, d\nu \tag{2}$$

where $\phi(\nu)$ is the filter transmission function (a number between zero and one). The magnitude system is calibrated[3] by specifying $F_{0,i}$ and $m_{0,i}$ which may be done in a number of ways. A common system is *Vega-magnitudes*, in which the star Vega is defined to have $m_{0,i} = 0$ (or something very close to zero) in all bands.

a) Grab the filter transmission functions and the spectrum of Vega[4] available on the website , and do a numerical integration to calculate the apparent V-band magnitude of Tycho's supernova, as he saw it. Assume it emitted a luminosity $L = 10^{43}$ ergs s^{-1} as a $T_{\rm SN} = 10000$ K blackbody and at a distance $D = 3$ kpc. Was this event easily visible to the naked eye?

Comment: This is one case in which the magnitude system is kind of helpful – the human eye can see things to about magnitude 6, and the north star is magnitude ≈ 2, Venus ≈ -4, the full moon ≈ -13. A distant galaxy (redshift $z \sim 1$) may be magnitude ~ 28. Given that Tycho's reported the event was about as bright as Venus, it is appears that dust extinction may have made the supernova appear ~ 2 magnitudes dimmer.

b) What is the apparent V-band magnitude of our light echo?

Comment Your calculations suggest that these light echoes are dim, but detectable. To take a spectrum will require multiple hours of exposure on a 10 meter class telescope like Keck. Such observations

[3] Criticizing the magnitude system is a bit of an astronomical cliche. For me, I like the $\log_{10}$, I can deal with the factor of 2.5, and the minus sign is kind of an in-joke. What I don't like is the offsets. As defined, a color of $m_B - m_V = 0$ does *not* mean that an object emits nearly equal fluxes in the B and V bands. It means that the B and V band fluxes are in the same ratio as they are for Vega, roughly a $T_{\rm eff} \approx 10^4$ K blackbody (in other words, rather blue). That is not very cool.

[4] Note that the Vega spectrum is given as F_λ with units ergs s^{-1} cm^{-2} Å^{-1}.

were made by [Krause et. al 2008](#) , who obtained a beautiful spectrum of Tycho's supernova. The observations of the echo matched perfectly the spectrum a normal Type Ia supernova taken a few weeks after the explosion (Figure 3), which definitively answered what kind of event it was. It is pretty amazing that we can revisit a great astronomical moment of the past, this time with 400 more years of technological advance in our hands.

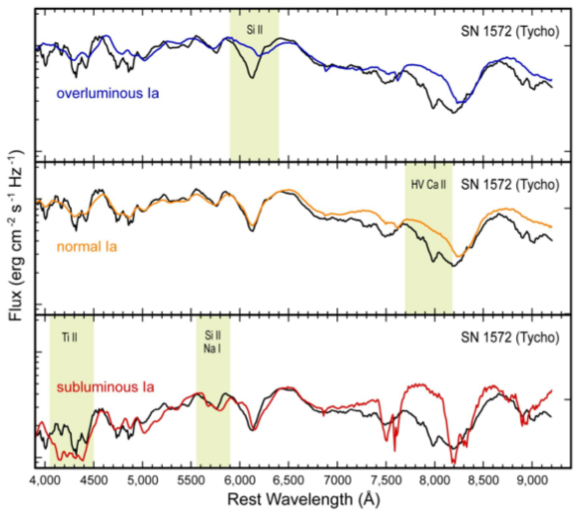

Figure 3: Spectroscopic observations of a light echo of Tycho's supernova, compared to spectra of Type Ia supernovae. From [Krause et. al 2008](#)

Your Beaming Face (Bonus, optional question)

WE'VE ALL HAD THE EXPERIENCE of being lost in a parking lot, trying to find our car by clicking the key remote repeatedly and hoping to hear a beep or see a flash. Well, now an old friend from high school presents the following unlikely [life hack](#) : **Hold your car remote against your chin when you push the button and you will increase its range.**

That sounded ridiculous, so I tried it myself. I found the maximum distance at which my remote would work (about 30 feet) then I held the remote to my chin and took several steps back. It worked! I was able to get several feet more range. Try it yourself!

So I had to hand it to my friend. Still, his explanation of the effect ("the water in your head acts like a huge radio antennae", boosting the signal) seemed pretty bogus to me. What do you think the *real* explanation for the extended range is? What is the theoretical maximum factor by which you might boost your range using this kind of trick?